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PSEUDOSCALAR-SCALAR TRANSITION FORM FACTORS IN COVARIANT LIGHT FRONT DYNAMICS

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In an explicitly covariant light-front formalism, we analyze transition form factors between pseudoscalar and scalar mesons. Application is performed in case of the $B \rightarrow f_0(980)$ transition in the full available transfer momentum range q^2 .

I. COVARIANT LIGHT FRONT FORMALISM

In the past few years, a generalization of the standard Light-Front Dynamics (LFD) has been proposed: Covariant Light-Front Dynamics[1] (CLFD). The formulation has already been successfully applied to relativistic particle and nuclear physics and it is particularly useful for describing hadrons, and all observables related to them, within the constituent quark model. In CLFD[1], the state vector which describes the physical bound state is defined on the light-front plane given by the equation $\omega \cdot r = \sigma$, where ω is an unspecified light-like four vector ($\omega^2 = 0$) which defines the position of the light-front plane and r is a four vector position of the system.

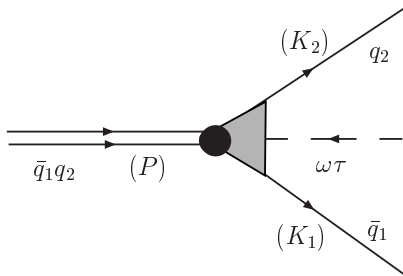


FIG. 1: Representation of the two body wave function on the light front.

CLFD proposes a formulation in which the evolution for a given system is expressed in terms of covariant expressions. Any four vector describing a phenomena can be transformed from one system of reference to another one by using a unique standard matrix which depends only on kinematical parameters and on ω . The particle is described by a wave function expressed in terms of Fock components of the state vector which respects the properties required under any transformation. The state vector describing a meson of momentum p , defined on a light-front plane characterized by ω , is given by:

$$|p, \lambda\rangle_\omega = (2\pi)^{3/2} \int \Phi_{j_1 \sigma_1 j_2 \sigma_2}^{J\lambda}(k_1, k_2, p, \omega\tau) a_{\sigma_1}^\dagger(\mathbf{k}_1) a_{\sigma_2}^\dagger(\mathbf{k}_2) |0\rangle \times \delta^{(4)}(k_1 + k_2 - p - \omega\tau) \exp(i\tau\sigma) 2(\omega \cdot p) d\tau \frac{d^3 k_1}{(2\pi)^{3/2} \sqrt{2\varepsilon_{k_1}}} \frac{d^3 k_2}{(2\pi)^{3/2} \sqrt{2\varepsilon_{k_2}}}, \quad (1)$$

where $\varepsilon_{k_i} = \sqrt{\mathbf{k}_i^2 + m_i^2}$ and \mathbf{k}_i is the momentum of the quark i . In Eq. (1) λ is the projection of the total angular momentum, J , of the system on the z axis in the rest frame and σ_1, σ_2 are the spin projections of the particles 1 to 2 in the corresponding rest systems. From the delta function ensuring momentum conservation, one gets:

$$\mathcal{P} = p + \omega\tau = k_1 + k_2. \quad (2)$$

To keep track of this conservation law, a momentum, $\omega\tau$, is assigned to the spurion but there is no fictitious particle in the physical state vector (see Fig. 1). We emphasize that the bound state wave function is always an off-energy shell object ($\tau \neq 0$ due to binding energy) and depends on the light-front orientation. The parameter τ

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is entirely determined by the on-mass shell condition for the individual constituents. The two-body wave function $\Phi^{J\lambda}(k_1, k_2, p, \omega\tau)$ written in Eq. (1) can be parametrized in terms of various sets of variables. We shall use in the following the usual light-front coordinates (x, \mathbf{R}_\perp) , which are defined by analogy to the equal time function in the infinite momentum frame as:

$$x = \frac{\omega \cdot k_1}{\omega \cdot p},$$

$$R_1 = k_1 - xp,$$

and where R_1 is decomposed in its spatial components parallel and perpendicular to the direction of the light-front, $R_1 = (R_0, \mathbf{R}_\perp, \mathbf{R}_\parallel)$. We have by definition $R_1 \cdot \omega = 0$, and thus $R_1^2 = -\mathbf{R}_\perp^2$.

II. SCALAR PARTICLE IN LIGHT FRONT

The explicit covariance of our approach allows us to write down the general structure of the two-body bound state. For a scalar particle composed of an antiquark and a quark of same mass, m , it has the form:

$$\Phi = \frac{1}{\sqrt{2}} \bar{u}(k_2) \left[\frac{A(\mathbf{k}^2)}{m} \right] v(k_1), \quad (3)$$

where $v(k_1)$ and $\bar{u}(k_2)$ are the usual antiparticle and particle Dirac spinors, and $A(\mathbf{k}^2)$ is the scalar component of the meson wave function, Φ . The expression for $A(\mathbf{k}^2)$ is as follows:

$$A(\mathbf{k}^2) = \frac{m}{2\varepsilon_k} g(\mathbf{k}^2), \quad (4)$$

where one defines $\varepsilon_k = \sqrt{\mathbf{k}^2 + m^2}$. The component $g(\mathbf{k}^2)$ will be parametrized by a gaussian wave function written as $g(\mathbf{k}^2) = 4\pi^2 \alpha \beta \exp(-\beta \mathbf{k}^2)$ where α and β are parameters to be determined from experimental data and theoretical assumptions. In terms of the variables (x, \mathbf{R}_\perp) , we have for the relative momentum between two quarks of same masses:

$$\mathbf{k}^2 = \frac{\mathbf{R}_\perp^2 + m^2}{4x(1-x)} - m^2, \quad (5)$$

where the relativistic, relative momentum, \mathbf{k} , corresponds, in the frame where $\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{0}$, to the usual relative momentum between the two particles.

III. PSEUDOSCALAR-SCALAR ($P \rightarrow S$) TRANSITION FORM FACTORS

In the Covariant Light-Front Dynamics formalism, the exact transition amplitude does not depend on the light front orientation. However, in any approximate computation the dependence is explicit and we can parametrize this dependence since our formalism is covariant. Hence,

the approximate amplitude expressed in CLFD is given by the following hadronic matrix,

$$\langle S(P_2) | \gamma^\mu \gamma^5 | P(P_1) \rangle = (P_1 + P_2)^\mu f_+(q^2) + (P_1 - P_2)^\mu f_-(q^2) + B(q^2) \omega^\mu, \quad (6)$$

where $B(q^2)$ is a non-physical form factor which has to be zero in any exact calculation. The last term in Eq. (6) represents the explicit dependence of the amplitude on the light front orientation ω . In order to extract the physical form factor $f_\pm(q^2)$, without any dependence on ω , from the amplitude $\langle S(P_2) | J^\mu | P(P_1) \rangle$, we will proceed as follow. First, we calculate the scalar products \mathcal{X}, \mathcal{Y} and \mathcal{Z} which are defined by,

$$\mathcal{X} = (P_1 + P_2)_\mu \cdot \langle S(P_2) | J^\mu | P(P_1) \rangle = f_+(q^2) [2(M_1^2 + M_2^2) - q^2] + f_-(q^2)(M_1^2 - M_2^2) + B(q^2) P_1 \cdot \omega (1 + y), \quad (7)$$

$$\mathcal{Y} = (P_1 - P_2)_\mu \cdot \langle S(P_2) | J^\mu | P(P_1) \rangle = f_-(q^2) q^2 + f_+(q^2)(M_1^2 - M_2^2) + B(q^2) P_1 \cdot \omega (1 - y), \quad (8)$$

with the meson masses, M_i , and

$$\mathcal{Z} = \frac{\omega_\mu \cdot \langle S(P_2) | J^\mu | P(P_1) \rangle}{\omega \cdot P_1} = f_-(q^2)(1 - y) + f_+(q^2)(1 + y). \quad (9)$$

In Eqs. (7, 8, 9) the term y defines the ratio between the two momenta P_1 and P_2 times the light-like four vector ω as,

$$y = \frac{\omega \cdot P_2}{\omega \cdot P_1} = \frac{M_2^2 + P_1 \cdot P_2}{M_1^2 + P_1 \cdot P_2}, \quad (10)$$

with $P_1 \cdot P_2 = \frac{1}{2}(M_1^2 + M_2^2 - q^2)$. For $q^2 > 0$, it is convenient to restrict ourselves to the plane defined by $\omega \cdot q = 0$. This condition is allowed in the system of reference where $\mathbf{P}_1 + \mathbf{P}_2 = \mathbf{0}$ with $P_{10} - P_{20} \neq 0$. From the scalar products \mathcal{X}, \mathcal{Y} and \mathcal{Z} we can isolate the form factors $f_\pm(q^2)$ from $B(q^2)$. Then, one gets the expressions for the form factors:

$$f_\pm(q^2) = \Omega(y, q^2) \Psi_\pm(y, q^2, \mathcal{X}, \mathcal{Y}, \mathcal{Z}), \quad (11)$$

where $\Omega(y, q^2)$ is identical for both form factors $f_\pm(q^2)$:

$$\Omega(y, q^2) = \frac{1}{4[(y-1)M_1^2 + q^2]y - M_2^2(y-1)}, \quad (12)$$

and where the functions $\Psi_\pm(y, q^2, \mathcal{X}, \mathcal{Y}, \mathcal{Z})$ have the following forms:

$$\Psi_-(y, q^2, \mathcal{X}, \mathcal{Y}, \mathcal{Z}) = \mathcal{Y}(y+1)^2 + \mathcal{X}(y^2-1) + [(1-3y)M_1^2 - M_2^2(y-3) + q^2(y-1)]\mathcal{Z},$$

and

$$\begin{aligned} \Psi_+(y, q^2, \mathcal{X}, \mathcal{Y}, \mathcal{Z}) = & \\ & \mathcal{Y}(y^2 - 1) + \mathcal{X}(y - 1)^2 + \\ & \left[(y - 1)M_1^2 - M_2^2(y - 1) + q^2(y + 1) \right] \mathcal{Z} . \end{aligned} \quad (13)$$

The second step is to express the amplitude $\langle S(P_2) | J^\mu | P(P_1) \rangle$ without using the form factors $f_\pm(q^2)$. In CLFD the leading contribution to the transition amplitude $\langle S(P_2) | J^\mu | P(P_1) \rangle$ is given by the diagram shown in Fig. 2. By using the CLFD rules, one can derive the matrix elements from the diagram given in Fig. 2 and one has,

$$\begin{aligned} \langle S(P_2) | J^\mu | P(P_1) \rangle = & \\ & \int \frac{d^2 \mathbf{R}_\perp dx d\theta}{2x(1-x)(2\pi)^3} \frac{1}{1-x'} \text{Tr} \left[-\bar{\vartheta}_s(m_1 + \not{k}_1) \right. \\ & \left. \gamma^\mu \gamma^5 (m_2 + \not{k}_2) \vartheta_p(m_3 - \not{k}_3) \right] , \end{aligned} \quad (14)$$

where ϑ_j is defined by:

$$\vartheta_p \propto \frac{1}{\sqrt{2}} A_p(\mathbf{k}^2) \gamma^5 , \vartheta_s \propto \frac{1}{\sqrt{2}} A_s(\mathbf{k}^2) . \quad (15)$$

The indices $i = p, s$ of ϑ_i denote the wave functions referring to the initial pseudoscalar and final scalar mesons, respectively. Note that x and x' are the fraction of the momentum carried by a quark q_3 (spectator quark) as given by:

$$x = \frac{\omega \cdot k_3}{\omega \cdot P_1} , \quad \text{and} \quad x' = \frac{\omega \cdot k_3}{\omega \cdot P_2} . \quad (16)$$

Now, one can replace the hadronic matrix element $\langle S(P_2) | J^\mu | P(P_1) \rangle$, which appears in the scalar products $\mathcal{X}, \mathcal{Y}, \mathcal{Z}$ defined in Eqs. (7, 8, 9), by the hadronic matrix elements $\langle S(P_2) | J^\mu | P(P_1) \rangle$ calculated by applying the CLFD diagrammatic rules and given in Eq. (14). Hence, by using Eq. (11) we are able to compute the form factors $f_\pm(q^2)$ as a function of q^2 and this over the whole available momentum range $0 < q^2 < q_{max}^2$.

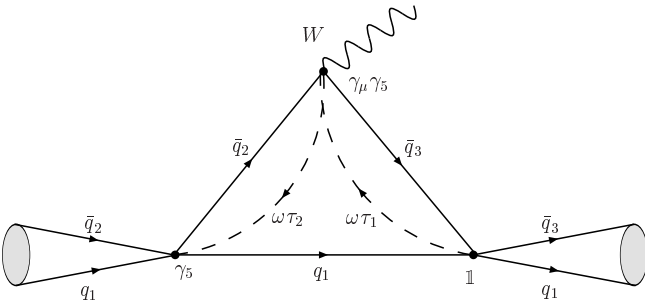


FIG. 2: Leading contribution in pseudoscalar-scalar transition.

IV. NUMERICAL APPLICATION

Introducing another set of form factors $F_0(q^2)$ and $F_1(q^2)$, the relationship between the two sets of form factors is

$$\begin{aligned} F_1(q^2) &= -f_+(q^2) , \\ F_0(q^2) &= -f_+(q^2) - \frac{q^2}{M_1^2 - M_2^2} f_-(q^2) . \end{aligned} \quad (17)$$

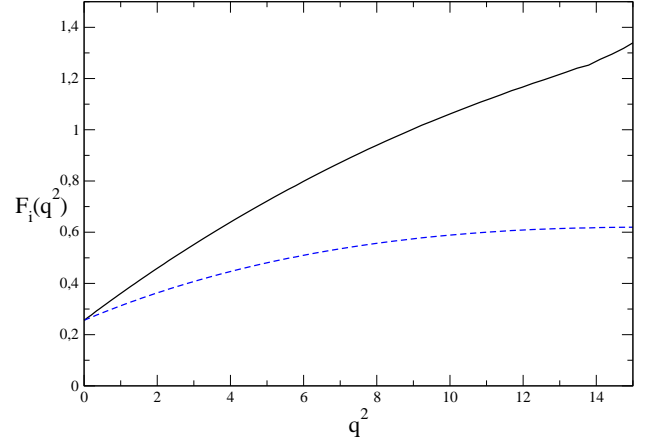


FIG. 3: Transition form factors $F_0(q^2)$ (dashed line) and $F_1(q^2)$ (full line) given in case of $B^u \rightarrow f_0(980)^u$.

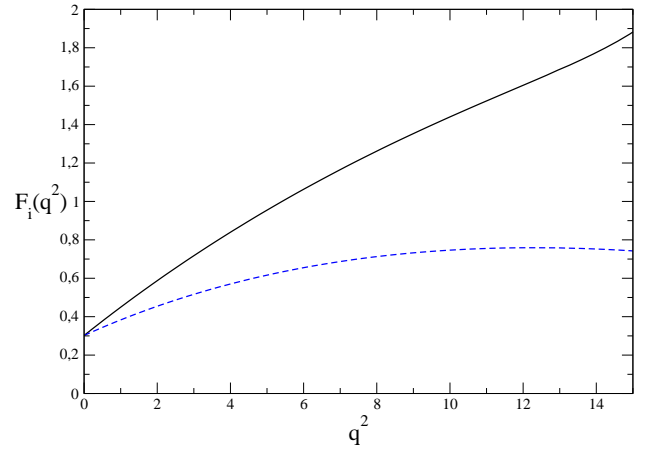


FIG. 4: Transition form factors $F_0(q^2)$ (dashed line) and $F_1(q^2)$ (full line) given in case of $B^s \rightarrow f_0(980)^s$.

Note that at $q^2 = 0$, one obtains $F_1(q^2 = 0) = F_0(q^2 = 0) = -f_+(q^2 = 0)$. We calculated the transition form factors in the case of the pseudoscalar-scalar transitions, such as $B^u \rightarrow f_0(980)$ and $B^s \rightarrow f_0(980)$. The weak transitions mentioned previously are induced by a current quark transition, $b \rightarrow u$ or $b \rightarrow s$ according to which component of the $f_0(980)$ one focuses on. We are working

in a constituent quark model where the CLFD formalism is applied. The wave functions used to describe the particles B^u, B^s and $f_0(980)$ have been determined using the same approach as that for the form factors and have been constrained by physical observable such as decay constant and the normalization as well[2]. In our model of the scalar meson $f_0(980)$, one assumes it is made of components $u\bar{u}, d\bar{d}$ and $s\bar{s}$ with constraints from D branching ratios. All the details regarding its phenomenological determination can be found in paper[2]. The results are shown in Figs. 3 and 4 where the form factors $F_0(q^2)$

and $F_1(q^2)$ are plotted as a function of the momentum transfer q^2 . Note that the theory allows us to directly obtain the behaviour of the form factors for all values of q^2 without extrapolation. Through the form factors $F_i(q^2)$ (that depend on the wave functions B and $f_0(980)$), derived within the CLFD formalism, the hadronic matrix elements that drives the weak decay transition between two hadronic states can be known more precisely. The better we describe electroweak decay transitions, the better we will be able to understand quark flavour changing within the Standard Model.

[1] J. Carbonell, B. Desplanques, V. A. Karmanov and J. F. Mathiot, "Explicitly covariant light-front dynamics and relativistic few-body Phys. Rept. **300** (1998) 215 [arXiv:nucl-th/9804029].

[2] B. El-Bennich, O. Leitner, J. P. Dedonder and B. Loiseau, paper in preparation.